The effect of time synchronization errors on the performance of cooperative MISO systems

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***Abstract*— We consider a wireless sensor network scenario where closely packed nodes can be grouped into clusters. Although each node might have only a single antenna, nodes within a cluster can cooperate during transmission or reception, thereby forming a cooperative Multiple Input Multiple Output (MIMO) system. Much of the work in the area of cooperative MIMO has assumed perfect synchronization between the nodes in the network and the associated benefits have been investigated. We examine this assumption in a system where a cluster of nodes cooperatively transmits to a single receive node over a flat fading channel. We evaluate the effect of clock jitter between nodes on the performance of this cooperative Multiple Input Single Output (MISO) system. The clock jitter at the transmit nodes results in the lack of a reference clock at the receive node and causes inter-symbol interference (ISI). This leads to a performance degradation in the system. However, simulation results indicate that jitters as large as 10% of the bit time do not have much effect on the BER performance of the system. We also show that when the channel is not undergoing fading, the power penalty due to the clock jitter is independent of the number of transmit nodes and the penalty increases as the SNR increases.**

I. INTRODUCTION

In large wireless sensor networks, the distribution of nodes can be looked at in terms of clusters where nodes that are physically close are grouped together [1], [2]. Within a cluster, nodes can communicate with relatively low power and cost as compared to inter-cluster communication. Typically, nodes in a wireless network will be equipped with a single antenna. However, we can use nodes within a cluster in order to form an equivalent multiple antenna system. We can obtain spatial diversity, reduce the energy consumption [3], [4], and increase capacity [5] using this cooperative multiple input multiple output (MIMO) system.

Previous work in the area of cooperative MIMO has assumed that the nodes are synchronized during transmission and reception. Our work differs from the previous work in that we assume a perfect noiseless channel between nodes within a cluster. However, we allow clock synchronization errors to be present between nodes in a cluster. Although we can implement synchronization algorithms [6], [7] to have very fine synchronization within each cluster, we will typically have to expend more energy in order to obtain a greater degree of synchronization. Therefore, there is a trade-off between synchronization accuracy and energy efficiency.

A recent work by Barriac *et al.* [8] investigated the effect of sensor placement errors on the performance of distributed beamforming schemes. In our work, we assume perfect carrier synchronization while time jitter may be present in a more general sense and need not occur due to placement errors alone. The carrier synchronization can be achieved by transmitting a reference carrier and all the nodes can lock to this reference carrier using a phase locked loop. We quantify the effect of the inter-symbol interference (ISI) caused due to the time jitters on the performance of a cooperative multiple input single output (MISO) system. The cooperation between the nodes within each cluster also entails some bandwidth and power penalty. This penalty is a subject for future investigation and in this paper we quantify the effect of the synchronization errors alone. The results indicate that both the cooperative transmit maximal ratio combining (MRC) [9] and Alamouti [10] schemes have good jitter tolerance and hence the synchronization algorithm can be made simple and energy efficient as fine time synchronization between the nodes is not required.

Winters [11] investigated the performance of a transmit diversity scheme in a MISO system with wired nodes where intentional delays are introduced at the transmitter in order to create frequency selective fading at the receive node. Thus, frequency diversity is obtained. Our work is different as we consider cooperative MISO schemes that have spatial diversity at the transmitter and the delays present in these schemes are due to the lack of synchronization and are not intentional.

In a cooperative single input multiple output (SIMO) system, a single node broadcasts messages to the receive cluster. Nodes in the receive cluster exchange their bits or pool information into the clusterhead by means of handshaking. Since this can be done without the need for time synchronization, the system performance is not affected by clock jitter. Hence, the SIMO system is not investigated further in this paper.

The remainder of the paper is organized as follows. The system model for the cooperative MISO system is described in Section II. We analyze the effect of clock jitter on the system performance in Section III and numerical results are given in Section IV. The simulation results are discussed in Section V. Finally, some concluding remarks are given in Section VI.

II. THE SYSTEM MODEL

Consider the system in Fig. 1 consisting of a cluster of M_T nodes that cooperatively transmit information to a single receive node. Although each node has only a single antenna, we can form a virtual MISO system through cooperation among nodes within the transmit cluster. Nodes within a cluster can exchange their data by means of handshaking and decide on the bit to be transmitted by each node in each time slot. We assume a perfect channel among nodes in a cluster

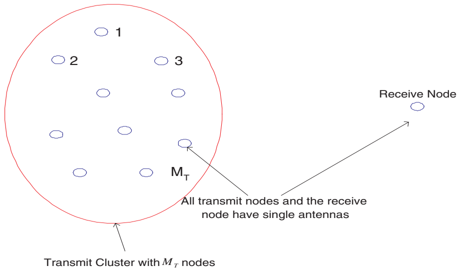

Fig. 1. The cooperative MISO system.

except for the synchronization error between the clocks and we ignore interference from the neighboring clusters in order to study the effect of time jitter alone.

We denote the reference clock for the transmit cluster as T_0. The clock jitter for each transmit node from the reference clock is denoted by $T_j^{(m)}$ where $1 \leq m \leq M_T$. We assume that the algorithm used to periodically synchronize nodes within a cluster leads to a maximum jitter of $\pm\frac{\Delta T_b}{2}$ around the reference clock T_0 for each node. Neglecting any drifts in the node clocks, the time jitters are fixed between any two runs of the synchronization algorithm while they can vary within the maximum allowable limits from one inter-synchronization period[1] to another. Therefore, when we are interested in the average performance of the system over many synchronization runs, the clock jitter of each node will have a certain distribution (uniform, gaussian, etc.) between $[-\frac{\Delta T_b}{2}, \frac{\Delta T_b}{2}]$. We can also investigate the worst case performance where the clock jitter can be fixed at certain values. When the transmit cluster has only 2 nodes, the worst case corresponds to fixing the jitter at the extreme ends, i.e. $T_j^{(1)} = -\frac{\Delta T_b}{2}, T_j^{(2)} = +\frac{\Delta T_b}{2}$. In a distributed MISO system, the effect of the time jitter at the transmit nodes is that the composite pulse shape (sum of the pulses from each node shifted by the corresponding time jitters) seen at the receiver will no longer be Nyquist[2]. Therefore, the neighboring bits will contribute ISI to the system and the performance of the system will degrade.

In order to analyze the performance of the system averaged over many synchronization runs, the clock jitter is assumed to be uniformly distributed between $[-\frac{\Delta T_b}{2}, \frac{\Delta T_b}{2}]$, where $0 \leq \Delta T_b \leq T_b$ and T_b is the bit duration. This distribution satisfies the requirement that synchronization algorithms typically bound the time jitter to a certain interval. The modulation is chosen to be uncoded BPSK[3] and raised cosine pulse shaping is employed. For the purpose of analysis and simulation, we restrict the raised cosine pulse to two sidelobes on either side since the sidelobes decay to more than 10 dB below the peak of the main lobe within this period. The total transmit energy for the cooperative system is denoted by E_s and is equal to the energy of the SISO system that we use for comparison.

The $M_T \times 1$ MISO channel is assumed to be undergoing frequency flat fading with Doppler frequency f_D and with independent fading on each of the M_T SISO links. Therefore, we can represent our M_T SISO channels as

$$h_m(t) = \alpha_m \delta(t), \; 1 \leq m \leq M_T \tag{1}$$

where α_m are the i.i.d fade variables.

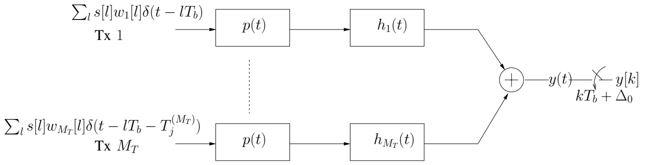

Fig. 2. The system block diagram for the transmit MRC scheme.

A. The transmit MRC scheme

The block diagram for transmit MRC scheme is shown in Fig. 2. The bit to be transmitted by all nodes during the l^{th} bit period is denoted by $s[l]$. The weighting factors at each node for the l^{th} bit period are given by

$$w_m[l] \;=\; \sqrt{M_T}\frac{\alpha_m[l]^*}{\sqrt{\|h[l]\|_F^2}}, \; 1 \leq m \leq M_T \tag{2}$$

where $\alpha_m[l]$ is the sample of the fade variable α_m at the l^{th} bit period. $\|h[l]\|_F^2$ is the Frobenius norm of the channel vector during the l^{th} bit period and is defined as

$$\|h[l]\|_F^2 = \sum_{m=1}^{M_T} |\alpha_m[l]|^2 \tag{3}$$

The channel estimates can be obtained by various methods such as feedback from the receiver or by using reciprocity [9]. The sampling offset at the receiver is denoted by Δ_0 and AWGN noise $n(t)$ is added at the front end of the receiver. The continuous time signal at the receive node is given by

$$y(t) = \sum_{l=-\infty}^{\infty} \sum_{m=1}^{M_T} \sqrt{\frac{E_s}{M_T}} s[l] w_m[l] \alpha_m[l] p(t - lT_b - T_j^{(m)}) \;+\; n(t) \tag{4}$$

The receiver samples the signal at time instants $t = kT_b + \Delta_0$ and using the formula for the MRC weights from (2), the resultant discrete time signal is given by

$$y[k] = \sum_{l=-\infty}^{\infty} \sqrt{\frac{E_s}{\|h[l]\|_F^2}} s[l] \sum_{m=1}^{M_T} |\alpha_m[l]|^2 p((k-l)T_b + \Delta_0 - T_j^{(m)}) \;+\; n[k] \tag{5}$$

We can express this discrete time received signal as a sum of the "desired signal", interference terms and noise.

$$y[k] = y_{desired}[k] + y_{isi}[k] + n[k] \tag{6}$$

For decoding the k^{th} bit, the "desired signal" is given by

$$y_{desired}[k] = \sqrt{\frac{E_s}{\|h[k]\|_F^2}} s[k] \sum_{m=1}^{M_T} |\alpha_m[k]|^2 p(\Delta_0 - T_j^{(m)}) \tag{7}$$

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[1] The time interval between two runs of the synchronization algorithm.

[2] The composite received pulse does not have zeros at time shifts in multiples of a bit time from the main lobe peak due to time jitter.

[3] It is anticipated that BPSK and QPSK will be the most common modulation schemes for sensor networks.

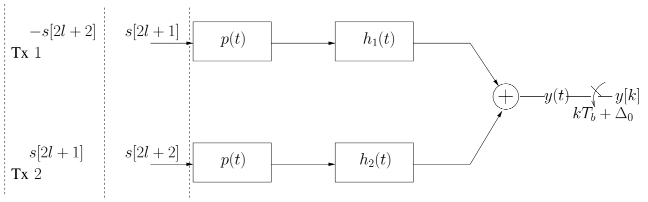

Fig. 3. The system block diagram for the 2 × 1 Alamouti scheme.

In the absence of time jitters ($T_j^{(m)} = 0$), (7) shows that the pulses from all the transmit nodes will perfectly overlap in time and add at the receiver. The sampling offset Δ_0 can then be chosen in order to obtain samples at the peak of the pulse. Due to the clock jitters, the pulses from the various transmit nodes will not overlap in time and irrespective of the choice of sampling offset Δ_0, the magnitude of the desired signal will be lower when compared to the zero jitter case. Using the fact that we have restricted the raised cosine pulse to two sidelobes on either side, we can express the ISI term as

$$y_{isi}[k] = \sum_{m=1}^{M_T} \sqrt{E_s} \sum_{\substack{l=k-3 \\ l\neq k}}^{k+3} \frac{|\alpha_m[l]|^2}{\sqrt{\|h[l]\|_F^2}} s[l] p((k-l)T_b + \Delta_0 - T_j^{(m)}) \tag{8}$$

B. The Alamouti scheme

The 2x1 Alamouti scheme is depicted in Fig. 3. We assume that the channel remains constant over two bit periods. Similar to the transmit MRC scheme, we sample the received signal $y(t)$ at the time instants $t = kT_b + \Delta_0$. We combine two consecutive received signal samples in order to determine the two transmitted bits by weighting the received samples with the appropriate channel gains. Thus, we perform standard Alamouti combining as described in [10]. Following an approach similar to Section II-A, we express the input to the decoder as

$$y[k] = y_{desired}[k] + y_{isi}[k] + n[k] \tag{9}$$

where the desired signal for decoding the $k = (2p+1)^{th}$ bit at the receiver is given by

$$y_{desired}[2p+1] = \sqrt{\frac{E_s}{2}} s[2p+1] \sum_{m=1}^{2} |\alpha_m[p]|^2 p(\Delta_0 - T_j^{(m)}) \tag{10}$$

The ISI terms can also be derived along similar lines as is done in Section II-A.

III. PERFORMANCE ANALYSIS OF TRANSMIT MRC

In this section we analyze the performance of the transmit MRC scheme over many synchronization runs. We assume that $\forall\ m$, $(\Delta_0 - T_j^{(m)})$ is uniformly distributed on $[-\frac{\Delta T_b}{2}, \frac{\Delta T_b}{2}]$. A piecewise linear approximation of the raised cosine pulse is chosen with slopes $\pm m_i$ as shown in Fig. 4 where $m_i > 0$ for all i. This approximation is tight for $0 \le \Delta T_b < 0.5T_b$ and is given by

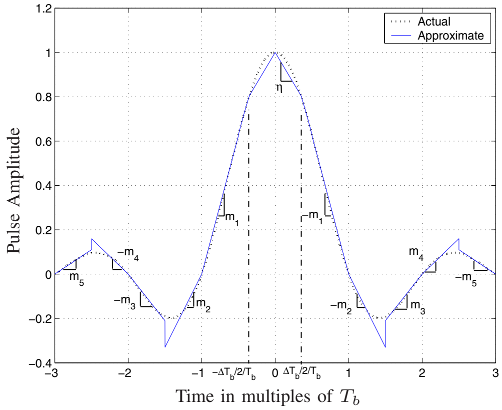

Fig. 4. The piecewise linear approximation of the raised cosine pulse.

TABLE I
COEFFICIENTS IN THE PIECEWISE LINEAR APPROXIMATION OF THE RAISED COSINE PULSE

l	$k-3$	$k-2$	$k-1$	$k+1$	$k+2$	$k+3$
m_l^+	0	m_4	$-m_2$	m_1	$-m_3$	m_5
m_l^-	$-m_5$	m_3	$-m_1$	m_2	$-m_4$	0

$$p((k-l)T_b + \Delta_0 - T_j^{(m)}) \approx \begin{cases} m_l^+ \times \frac{(\Delta_0 - T_j^{(m)})}{T_b}, & \Delta_0 - T_j^{(m)} > 0 \\ m_l^- \times \frac{(\Delta_0 - T_j^{(m)})}{T_b}, & \Delta_0 - T_j^{(m)} < 0 \end{cases} \tag{11}$$

where m_l^+ and m_l^- are related to the slopes m_i by Table I.

Assuming that the desired bit is $s[k] = 1$, the worst case scenario will occur when all the interference terms are negative. Since $m_i > 0$, we find that the worst case scenario will depend on the bits $s[l], k-3 \le l \le k+3,\ l \neq k$ and the sign of $(\Delta_0 - T_j^{(m)})$. We can obtain an upper bound to the worst case interference by taking the absolute value of the jitters. This will make all the jitters positive and the worst case interference for this situation is given by

$$y_{isi}[k] = -\sqrt{E_s} \sum_{m=1}^{M_T} \beta_m \frac{|(\Delta_0 - T_j^{(m)})|}{T_b} \tag{12}$$

where

$$\beta_m = \sum_{\substack{l=k-3 \\ l\neq k}}^{k+3} \frac{|\alpha_m[l]|^2}{\sqrt{\|h[l]\|_F^2}} |m_l^+| \tag{13}$$

Since $(\Delta_0 - T_j^{(m)}) \sim U[-\frac{\Delta T_b}{2}, \frac{\Delta T_b}{2}]$, we obtain $\beta_m \frac{|(\Delta_0 - T_j^{(m)})|}{T_b} \sim U[0, \beta_m \frac{\Delta T_b}{2T_b}]$. The sum of M_T uniform probability distributions converges to a Gaussian distribution very quickly for $M_T > 2$ and the interference terms are represented by a Gaussian random variable given by

$$y_{isi}[k] = \sqrt{E_s} \mathcal{N}(\mu_{isi}, \sigma_{isi}^2) \tag{14}$$

with its mean and variance respectively given by

$$\mu_{isi} = -\textstyle\sum_{m=1}^{M_T} \beta_m \frac{\Delta T_b}{4T_b}, \quad \sigma_{isi}^2 = \displaystyle\sum_{m=1}^{M_T} \beta_m^2 \frac{\Delta T_b^{\,2}}{48T_b^2} \tag{15}$$

In order to analyze the "desired signal", we can approximate the main lobe of the raised cosine pulse by a triangle for the interval $-\frac{\Delta T_b}{2} \leq t \leq \frac{\Delta T_b}{2}$ as shown in Fig. 4. This approximation can be analytically written as

$$p(\Delta_0 - T_j^{(m)}) \approx 1 - \eta \times \frac{|(\Delta_0 - T_j^{(m)})|}{T_b} \tag{16}$$

where

$$\eta = \frac{2T_b}{\Delta T_b}\left(1 - p\left(\frac{\Delta T_b}{2T_b}\right)\right) \tag{17}$$

This approximation is good for small values of clock jitter ΔT_b, however it is a bit loose for larger values of clock jitter and consequently, we estimate a lower value of the desired signal. We use this approximation in order to obtain a bound on the worst case signal to interference and noise ratio (SINR). We can show that for $M_T > 2$, the desired signal converges to a Gaussian random variable given by

$$y_{desired}[k] = \sqrt{\frac{E_s}{\|h[k]\|_F^2}}\mathcal{N}(\mu_{desired}, \sigma^2_{desired}) \tag{18}$$

with its mean and variance respectively given by

$$\begin{aligned} \mu_{desired} &= \sum_{m=1}^{M_T} |\alpha_m[k]|^2 \left(1 - \eta\frac{\Delta T_b}{4T_b}\right) \\ \sigma^2_{desired} &= \sum_{m=1}^{M_T} \eta^2 |\alpha_m[k]|^4 \frac{\Delta T_b^2}{48T_b^2} \end{aligned} \tag{19}$$

The noise in the system is assumed to be AWGN with power $N_0 B$ where $B\left(= \frac{1}{2T_b}\right)$ is the bandwidth occupied by the BPSK signal and N_0 is the one-sided power spectral density of the noise. Using (6), (14) and (18), we can express the received discrete time signal for worst case interference as

$$y[k] = \mu_{eff} + \mathcal{N}(0, \sigma^2_{eff}) \tag{20}$$

where the mean of the effective signal and the variance of the effective noise in units of energy are given by

$$\begin{aligned} \mu_{eff} &= \sqrt{\frac{E_s}{\|h[k]\|_F^2}}\mu_{desired} + \sqrt{E_s}\mu_{isi} \\ \sigma^2_{eff} &= \frac{E_s}{\|h[k]\|_F^2}\sigma^2_{desired} + E_s\sigma^2_{isi} + \frac{N_0}{2} \end{aligned} \tag{21}$$

From the above analysis, we observe that the effective mean of the signal is decreased both due to the ISI as well as the decrease in the desired signal's amplitude due to the clock jitter while the variance of the noise increases not only by the ISI but also due to the uncertainty of the desired signal.

The instantaneous SINR of the system is given by

$$\gamma(\bar{\boldsymbol{\alpha}}) = \mu^2_{eff}/\sigma^2_{eff} \tag{22}$$

where $\bar{\boldsymbol{\alpha}} = [\bar{\alpha}_1 \; \bar{\alpha}_2 \ldots \bar{\alpha}_{M_T}]$, $\bar{\alpha}_m = \left[\alpha_m[k{-}3] \ldots \alpha_m[k{+}3]\right]$ for $1 \leq m \leq M_T$.

In order to find the average SINR of the system, we average the instantaneous SINR over the fade variables of the M_T independent SISO links and over a period of seven consecutive bit times. Thus the average SINR of the system is given by

$$\bar{\gamma} = \underbrace{\int_0^\infty \int_0^\infty \ldots \int_0^\infty}_{7\times M_T - fold} \gamma(\bar{\boldsymbol{\alpha}}) \prod_{m=1}^{M_T} f\left(\bar{\alpha}_m\right) d\bar{\alpha}_1 d\bar{\alpha}_2 \ldots d\bar{\alpha}_{M_T} \tag{23}$$

Similarly, the expression for the average BER can be derived. In order to evaluate these expressions, we need to determine the expressions for the joint pdf $f\left(\bar{\alpha}_m\right)$ between 7 consecutive bit times of the fading process. Using this joint pdf, we can evaluate the expressions for the average SINR and average BER for the cooperative transmit MRC scheme.

IV. NUMERICAL RESULTS

We evaluate the expressions derived in Section III for the case of a static channel with equal gains on all antennas. Therefore, all our channel gains are set to 1 for all bit periods l, i.e.

$$|\alpha_m[l]| = 1, \; 1 \leq m \leq M_T \tag{24}$$

Using (15), (19), (21) and (22), we obtain the multiplicative reduction factor in the average SINR ($\bar{\gamma}$) due to the transmit clock jitter as

$$\Delta\bar{\gamma} = \left\{ \frac{\left(1 - \eta\frac{\Delta T_b}{4T_b} - \sum_{i=1}^{5} m_i \frac{\Delta T_b}{4T_b}\right)^2}{1 + \frac{2E_s}{N_0}\frac{\Delta T_b^2}{48T_b^2}\left(\eta^2 + (\sum_{i=1}^{5} m_i)^2\right)} \right\} \tag{25}$$

This translates into a power penalty for the system due to the transmit clock jitter. The power penalty is independent of the number of transmit nodes and is a function of the SNR ($\frac{E_s}{N_0}$) and the fractional transmit clock jitter only.

The power penalty is plotted in Fig. 5 for different values of clock jitter and SNR. From the plot, we observe that the power penalty is larger when the SNR is higher. This result seems counter-intuitive as we would expect a system with higher SNR to have better jitter tolerance. However, careful observation reveals that the power penalty will be more for higher SNR as the ISI caused by the clock jitter will have a more dominant effect for high SNRs than in the low SNR regime. Although the average SINR for the case of high $\frac{E_s}{N_0}$ will be greater than the average SINR for low $\frac{E_s}{N_0}$, the power penalty will follow the opposite trend. We also observe that for a jitter of 10%, the power penalty is only 0.7 dB.

V. SIMULATION DETAILS AND RESULTS

A. Determining the sampling offset at the receive node

In a cooperative MISO system, we effectively lose the transmit clock reference (T_0) at the receiver due to the transmit clock jitter. A question arises as to what the ideal sampling offset at the receiver should be in such a situation. For the purpose of simulation, in order to determine the sampling offset at the receiver, we adopt the following approach. We transmit a known preamble on each of the antennas. The receiver samples this signal at a very high rate (1000 samples per bit time T_b), performs a correlation with the known

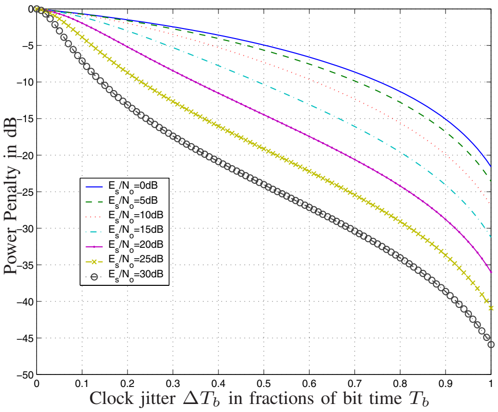

Fig. 5. The power penalty versus fractional clock jitter for various SNRs.

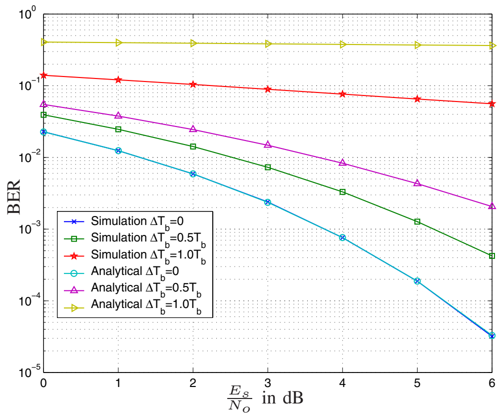

Fig. 6. BER analysis and simulation for static equal-gain channel.

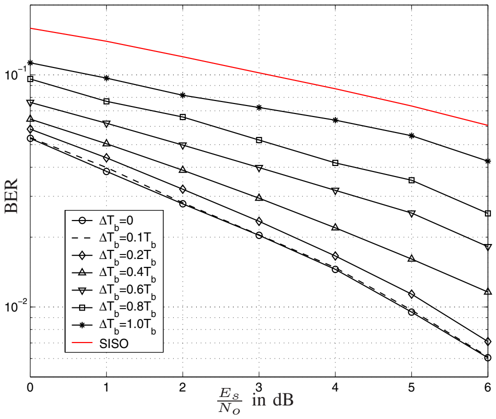

Fig. 7. The average BER for the 2×1 transmit MRC scheme.

preamble and then detects the offset that results in a peak in the correlation. This gives us our required sampling offset[4].

B. Simulation description

The 2x1 and 4x1 cooperative MISO systems were simulated using MATLAB and the effect of transmit clock jitter on the BER performance was studied for the transmit MRC (Fig. 2) and Alamouti (Fig. 3) schemes. A BPSK modulation scheme was chosen with a data rate of 250 Kbps. Raised cosine pulse shaping was employed with a roll-off factor $\alpha = 0.22$. The $M_T \times 1$ flat Rayleigh fading channel was simulated using Jake's model [12] with a doppler frequency (f_D) of 60 Hz. The BER calculations were performed by averaging over 10^5 bits. The average BER was calculated using 100 realizations of the time jitter corresponding to as many runs of the cluster synchronization algorithm. The sampling offset (Δ_0) at the receiver was calculated using the method outlined in Section V-A. The SNR range was limited to 0-6 dB due to simulation time constraints.

C. Simulation results

In the first simulation, the transmit MRC technique was employed for a MISO system over a static channel with equal gains on all antennas. Fig. 6 compares the simulation results for the worst case BER with the analysis presented in Section IV. The simulation results and analytical results match exactly for the zero jitter case. For higher values of jitter, the analytical results upper bound the results obtained via simulations. As expected, the analytical results give an upper bound on the BER of the cooperative MISO system.

Next, we simulated the performance of the system over a Rayleigh fading channel. The time jitters at the transmitters were uniformly distributed between $[-\frac{\Delta T_b}{2}, \frac{\Delta T_b}{2}]$. This gives us the average BER over a number of synchronization runs. Fig. 7 shows the BER versus SNR curves when the transmit MRC scheme is employed. We note that 10% jitter does not degrade the BER performance of the system. However, for jitters greater than 10%, the BER increases. The results also indicate that the 2x1 cooperative MISO system performs better than the SISO case even in the presence of 100% clock jitter. Therefore, the cooperative MISO system on average buys us some performance gains over a SISO system even in the presence of severe clock jitter. A 4x1 cooperative transmit MRC system was also simulated and once again we observed that 10% jitter does not affect the BER performance.

Fig. 8 is a plot of the BER performance of the transmit MRC scheme when worst case jitters are considered. For the 2x1 system, the jitters are set at $\pm\frac{\Delta T_b}{2}$. We again observe that 10% jitter does not have much effect on the performance. However, for higher SNRs (> 5 dB), we can notice a small degradation in the BER. The BER degrades much faster as the clock jitter ΔT_b increases as compared to the average performance and for a jitter of $70 - 80\%$, we notice that the SISO system starts outperforming the cooperative MISO system. Therefore, we can save the intra-cluster communication energy by not cooperating to transmit for a clock jitter greater than 80%. Moreover, for large clock jitter, we observe that the BER is almost constant for the simulated range of SNR values as the noise contributed by the clock jitter exceeds the AWGN noise.

The worst case BER performance for the 2x1 Alamouti scheme is plotted in Fig. 9. Although the BER does not degrade much for 10% jitter, the SISO system outperforms the Alamouti scheme for jitter greater than 50%. Similar to transmit MRC, we note that for high values of clock jitter, the BER is relatively flat for the simulated range of SNR values.

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[4] This is just an efficient method to estimate the ideal sampling offset for the purpose of simulations and is not meant to be a practical scheme.

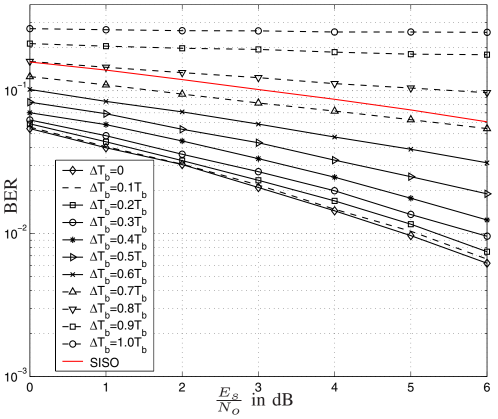

Fig. 8. The worst case BER for the 2×1 transmit MRC scheme.

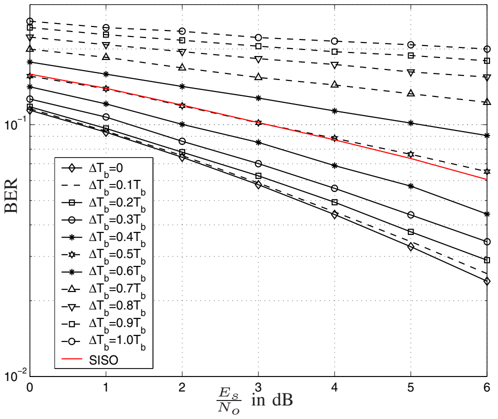

Fig. 9. The worst case BER for the 2×1 Alamouti scheme.

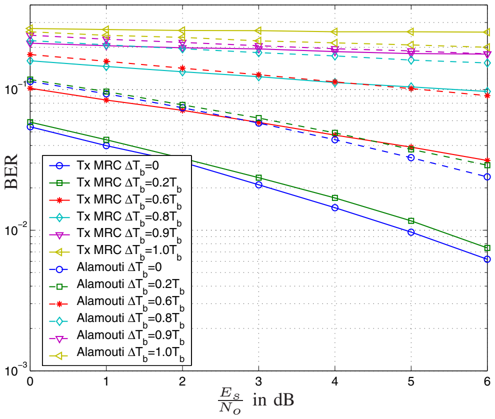

Fig. 10. Comparison of the 2×1 transmit MRC and Alamouti schemes.

Fig. 10 compares the worst case BER performance of the 2x1 transmit MRC and Alamouti schemes. We observe that for small values of clock jitter, the transmit MRC scheme easily outperforms the Alamouti scheme. The performance gap closes for large values of the clock jitter and for a jitter greater than $80 - 90\%$, the Alamouti scheme has a better worst case BER than the transmit MRC scheme. However, for such high values of clock jitter, a SISO system with the same transmit energy outperforms both cooperative MISO schemes and there is no benefit in using a cooperative transmission scheme.

We expect the power penalty of the cooperative MISO system due to time jitter, over a fading channel, to increase with SNR, similar to the results presented in section IV for static channels. This trend is visible in the plots presented in this section where the BER curve of the SISO system cuts across the BER curve of the cooperative MISO scheme with time jitter. Therefore, as the SNR increases, the SISO system will outperform the cooperative MISO system for lower values of the clock jitter. Consequently, the benefit of the cooperative MISO system with time jitter will reduce as the SNR increases.

VI. CONCLUSIONS

The effect of time synchronization errors in a cooperative MISO system was investigated. The clock jitter at the transmit cluster causes ISI, thereby reducing the mean of the received signal and increasing the variance of the noise. An analytic expression for the average SINR is derived that gives insight into how the clock jitter affects the system. The power penalty entailed in a static channel due to the transmit clock jitter turns out to be independent of the number of transmit nodes and is larger for higher SNRs.

Simulation results indicate that 10% jitter does not have much effect on the BER performance of the cooperative transmit MRC and Alamouti techniques. We find that a SISO system with the same transmit energy outperforms the worst case BER performance of the transmit MRC and Alamouti schemes for jitters greater than 80% and 50%, respectively, and there is no benefit in cooperative transmission for very large clock jitters. Since the cooperative MISO scheme has a good tolerance of up to 10% jitter, the synchronization algorithms can be made simpler and more energy efficient without sacrificing the performance of the system.

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